Isebrook School – Calculation Policy

Our calculation policy aims to develop all student's mathematical understanding at the same pace. As much as possible, children should be accessing the same learning. Differentiation should primarily be through support, scaffolding and deepening, not through task. Consistency in language is essential for pupils to understand the concepts presented in mathematics. If other, 'child-friendly' terminology is used, this must be alongside the current terminology recommended by maths specialists. Using this will support students with their examinations and throughout secondary school.

Concrete, pictorial, abstract (CPA) concepts should not be confused as differentiation for lower, middle, higher attaining students. CPA is an approach to be used with the whole class and teachers should promote each area as equally valid. Manipulatives in particular must not be presented as a resource to support the less confident or lower attaining pupils. The abstract should run alongside the concrete and pictorial stage as this enables pupils to better understand mathematical statements and concepts.

This policy will concentrate on the four basic operations in mathematics, addition, subtraction, multiplication and division. Rather than concentrating on the year groups covering topics, the policy is design to show the stages of teaching each topic and how it develops from concrete to pictorial to abstract. Each of the four operations build on a solid understanding of place value, the connections between the four number operations and number sense, such as: whether they are odd or even, whether they are close to multiples of ten or if they are close together.

- Students need to use correct mathematical terminology in context and be able to verbalise their calculation strategies.

- Students need to make considered decisions as to the most appropriate methods to make mathematics more functional. They need to choose the most appropriate, fluent, efficient and accurate method to do a particular calculation.

- Students need to use concrete resources before they progress to pictorial and abstract representations. This CPA (concrete, pictorial and abstract) approach needs to be available to children throughout school, as and when necessary. Use of manipulatives (numicon, Cuisenaire, dienes, HTO counters etc.) helps reinforce understanding and provides support when calculating mentally, mentally with jottings, using expanded methods and formal written methods. Use of the bar model, number lines and part-part whole diagrams are recommended.

- Students should progress between the stages working towards formal written methods (where appropriate), once they have mastered each stage. However, they should not be hurried and, after the method has been taught, children should still be able to make their preferred choice of the most appropriate, efficient and accurate method for them. Previous stages may need to be revisited to consolidate understanding when introducing a new strategy.

- As new methods of calculations are introduced, students should have the opportunity to examine them, alongside the method they have consolidated, to make connections between the methods and establish the similarities and differences between them.

Addition

Written methods for addition

It is important that student's mental methods of calculation are practised on a regular basis and secured alongside their learning and use of written methods of addition. The aim is that students use mental methods when appropriate, but for calculations that they cannot do in their heads they use a written method accurately and with confidence. Students are taught and acquire secure mental methods of calculation and one written method of calculation for addition which they know they can rely on when mental methods are not appropriate. This policy shows the possible stages of each written method for addition, each stage building towards a more refined method. There are some key basic skills that students need to help with addition, which include:

- counting
- estimating
- recalling all addition pairs to 10, 20 and 100 (7 + 3 = 10, 17 + 3 = 20, 70 + 30 = 100)
- knowing number facts to 10 (6 + 2 = 8)
- adding mentally a series of one-digit numbers (5 + 8 + 4)
- adding multiples of 10 (60 + 70) or of 100 (600 + 700) using the related addition fact, 6 + 7, and their knowledge of place value
- partitioning two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways (432 into 400 + 30 + 2 and also into 300 + 120 + 12)
- understanding and using addition and subtraction as inverse operations

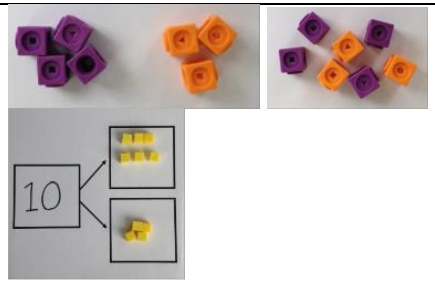
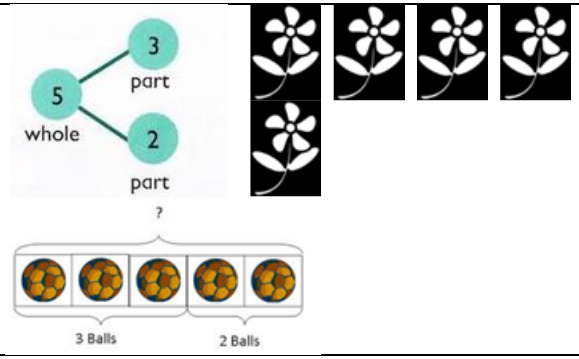
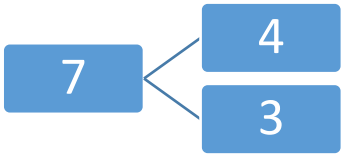
Using and applying is a key theme and one of the aims of National Curriculum and before children move onto the next stage in written calculation it is important that their skills are broadened through their use and application in a range of contexts, these include:

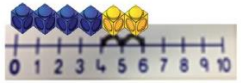
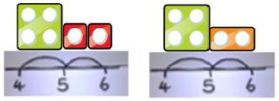
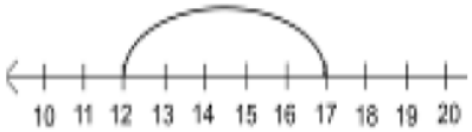
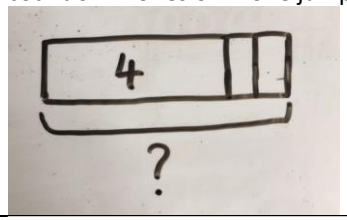
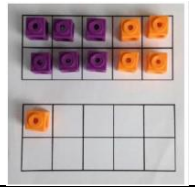
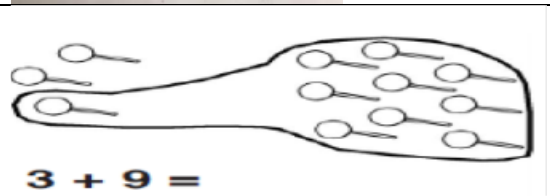
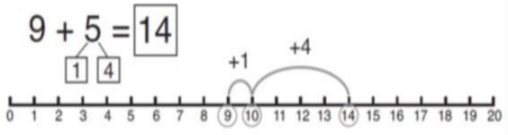
- using inverse
- missing box questions
- using units of measure including money and time
- word problems

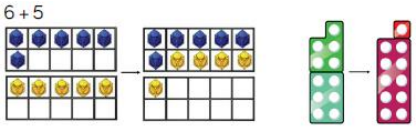
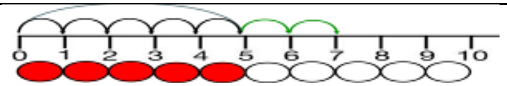
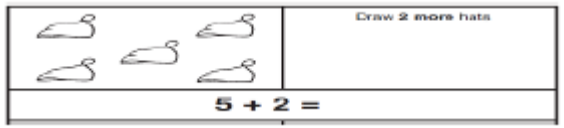
• open ended investigations

Stage 1: Practical (combining) and adding on (increasing)

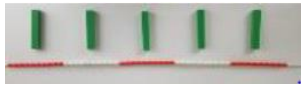
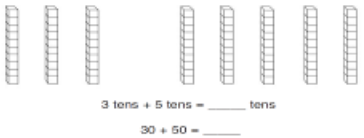
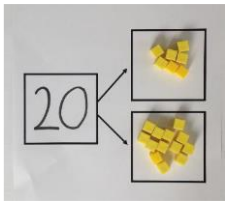
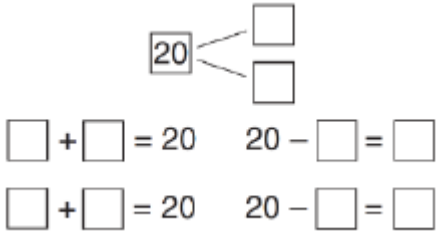
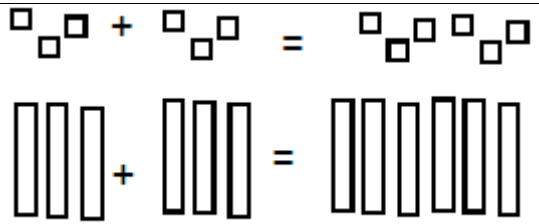
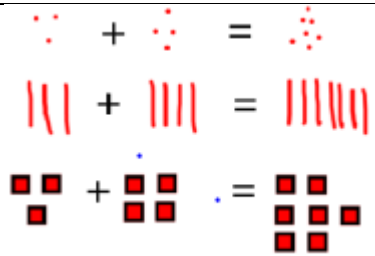
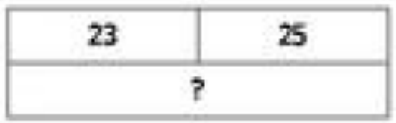
Prior to recording addition steps on a number line, students will work practically with equipment where they are combining sets of objects. As they become more confident, this practical addition of sets of objects will be mirrored on a number line so that the two are being done together and children are adding on. This will prepare them for the abstract concept of adding numbers rather than objects.

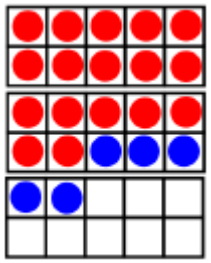
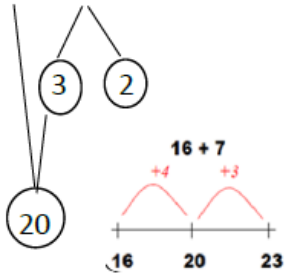
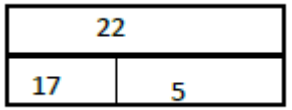
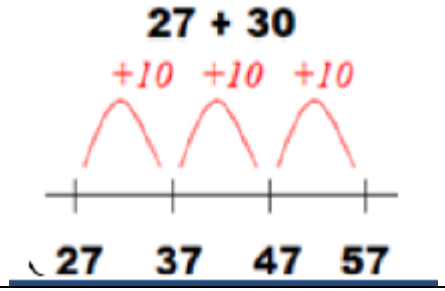
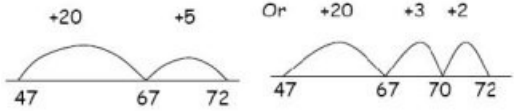
Objective and Strategy	**Concrete**	**Pictorial**	**Abstract**
Combining two parts to make a whole: part- whole model	 Use part of the whole model.		4 + 3 = 7 

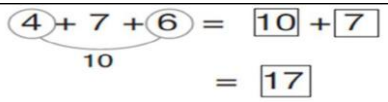
	Use cubes to add two numbers together as a group or in a bar. 	Use pictures to add two numbers together as a group or in a bar 	Use the part-part whole diagram as shown above to move into the abstract 10= 6 + 4
Starting at the bigger number and counting on	 Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.   Counting on using number lines using cubes or numicon	12 + 5 = 17  Start at the larger number on the number line and count on in ones or in one jump to find the answer.  A	5 + 12 = 17 Place the larger number in your head and count on the smaller number to find your answer.
Regrouping to make 10. *This is an essential skill for column addition later*	 6+5 =11 Start with the bigger number and use the smaller number to make 10  Use ten frames	 Use pictures or a number line. Regroup or partition the smaller number using the part, part whole model to make 10. 	7 + 4= 11 If I am at seven, how many more do I need to make 10. How many more do I add on now?

			
Represent & use number bonds and related subtraction facts within 20	 2 more than 5.	 	Include missing number questions: 8 = ? + 3 5 + ? = 8 Emphasis should be on the language ‘1 more than 5 is equal to 6.’ ‘2 more than 5 is 7.’ ‘8 is 3 more than 5.’

Stage 2 – Add numbers using concrete objects, pictorial representations and mental methods, bar modelling and number lines.

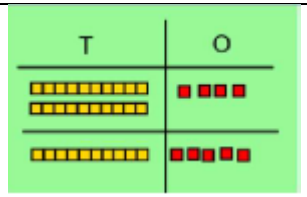
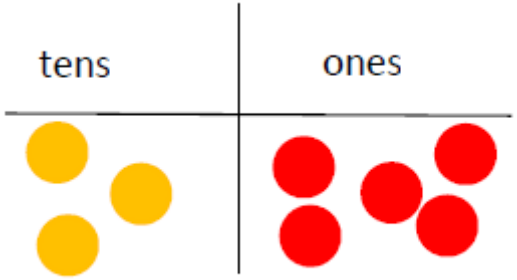
Objective and Strategy	Concrete	Pictorial	Abstract
Adding multiples of ten	50= 30 + 20  Model using dienes and bead strings	 3 tens + 5 tens = _____ tens 30 + 50 = _____ Use representations for base ten.	20 + 30 = 50 70 = 50 + 20 40 + □ = 60
Use known number facts Part, part, whole	 Children explore ways of making numbers within 20	 20 □ + □ = 20 20 – □ = □ □ + □ = 20 20 – □ = □	□ + 1 = 16 16 – 1 = □ 1 + □ = 16 16 – □ = 1
Using known facts		 Children draw representations of H,T and O	3+4 =7 Leads to 30 + 40 = 70 Leads to 300 + 400 = 700
Bar Modelling	 3 + 4 = 7	 7 + 3 = 10	 23 25 ? 23 + 25 = 48

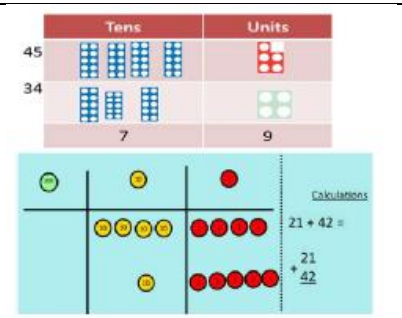
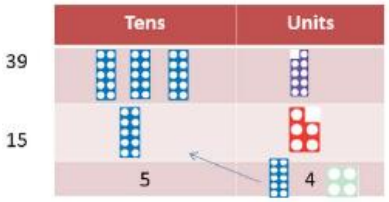
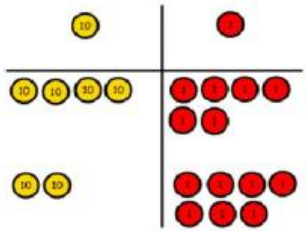
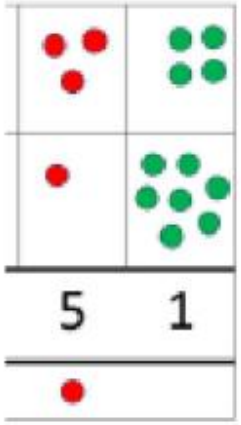
Add a two digits' number and ones	17 + 5 = 22  Use ten frame to make 'magic ten Children explore the pattern. 17 + 5 = 22 27 + 5 = 32	17 + 5 = 22  Use part, part, whole and number line to Model	17 + 5 = 22 Explore related facts 17 + 5 = 22 5 + 17 = 22 22—17 = 5 22—5 = 17 
Add a 2digit number and tens	 25 + 10 = 35 Explore that the ones digit does not change		27 + 10 = 37 27 + 20 = 47 27 + □ = 57
Add two 2-digit numbers	 Model using dienes, place value counters and numicon	 Use number line and bridge ten using part whole if necessary	25 + 47 20 + 5 40 + 7 20 + 40 = 60 5+ 7 =12 60 + 12 = 72

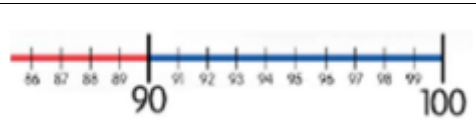
Add three 1-digit numbers	 Combine to make 10 first if possible, or bridge 1o then add third digit	 Regroup and draw representation 	 Combine the two numbers that make/bridge ten then add on the third.

Stage 3 -Partitioning (expanded columnar method)

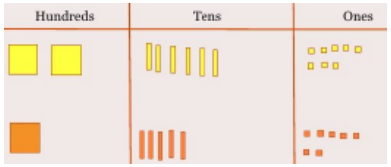
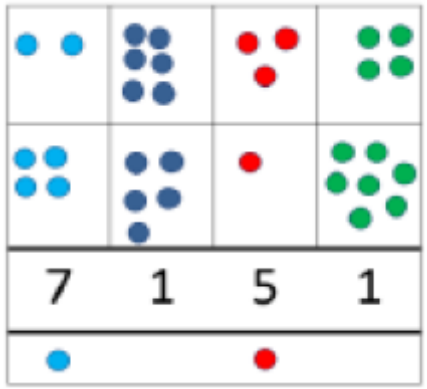
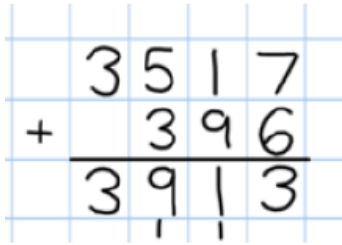
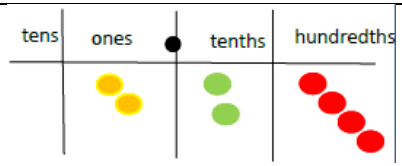
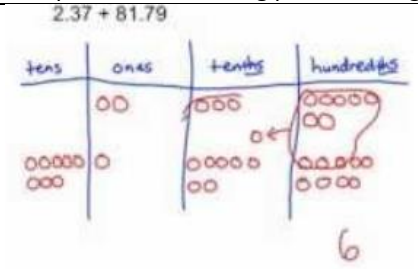
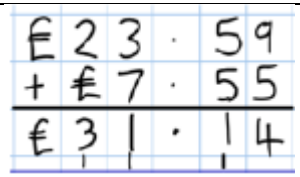
Partition both numbers into tens and units or hundreds, tens and units (using a grid makes this easier)

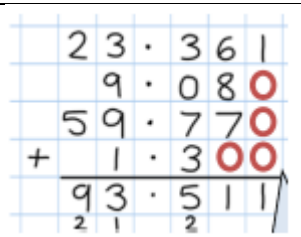
Objective & Strategy	Concrete	Pictorial	Abstract
Column Addition—no regrouping (friendly numbers) Add two or three 2 or 3digit numbers.	 Dienes or numicon Add together the ones first, then the tens.	Students move to drawing the counters using a tens and one frame. 	223 +114 337 Add the ones first, then the tens, then the hundreds.

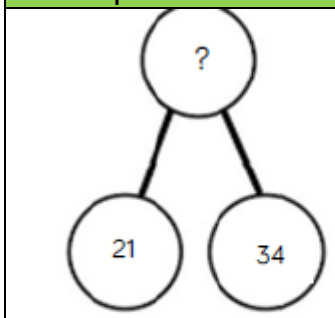
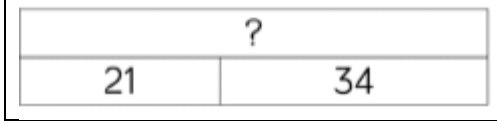
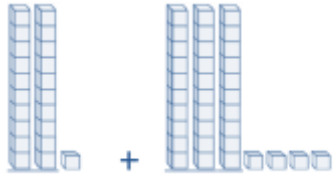
	 Move to using place value counters		
Column Addition with regrouping.	 Exchange ten ones for a ten. Model using numicon and place value counters.  46 + 27 = 73	 3 4 +1 7 5 1 Students can draw a representation of the grid to further support their understanding, carrying the ten **underneath** the line	20 + 5 40 + 8 60 + 13 = 73 Start by partitioning the numbers before formal column to show the exchange 536 +85 621 11
Estimate the answers to questions and use inverse operations to check answers		Use number lines to illustrate estimation.	Building up known facts and using them to

	Estimating 98 + 17 = ? 100 + 20 = 120		illustrate the inverse and to check answers: 98 + 18 = 116 116 – 18 = 98 18 + 98 = 116 116 – 98 = 18

Stage 4- Efficient (column method)

Objective & Strategy	Concrete	Pictorial	Abstract
Add numbers with up to 4 digits	Students continue to use dienes, numicon or place value counters to add, exchanging ten ones for a ten and ten tens for a hundred and ten hundreds for a thousand. 	 Draw representations using place value grid.	 Continue from previous work to carry hundreds as well as tens. Relate to money and measures.
Add decimals with 2 decimal places, including money.			

			 Insert zeros for place holders.

Conceptual variation; different ways to ask students to solve 21 + 34			
 	Word problems: In year 7 there are 21 students and in year 8 there are 34 students. How many students in total? 21 + 54 = 54 Prove it!	Calculate the sum 21 +34 21 +34 ____ 21 + 34 = □ = 21 + 34	 Missing digit problems 

Subtraction

Written methods for Subtraction

It is important that children's mental methods of calculation are practised on a regular basis and

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calculation for subtraction which they know they can rely on when mental methods are not appropriate. This policy shows the possible stages of each written method for subtraction, each stage building towards a more refined method.

There are some key basic skills that children need to help with subtraction, which include:

- counting
- estimating
- recalling all addition pairs to 10, 20 and 100 along with their inverses (7 + 3 = 10, 10 – 3 = 7,
17 + 3 = 20, 20 – 3 = 17, 70 + 30 = 100, 100 – 30 = 70)
- knowing number facts to 10 and their inverses (6 + 2 = 8, 8 - 2 = 6)
- subtracting multiples of 10 (160 - 70) using the related subtraction fact, 16 - 7, and their knowledge
of place value
- partitioning two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways
(432 into 400 + 30 + 2 and also into 300 + 120 + 12)
- understanding and using subtraction and addition as inverse operations

Using and applying is a key theme and one of the aims of National Curriculum and before students move onto the next stage in written calculation it is important that their skills are broadened through their use and application in a range of contexts, these include:

- using inverse
- missing box questions
- using units of measure including money and time
- word problems

- open ended investigations

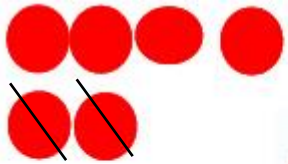
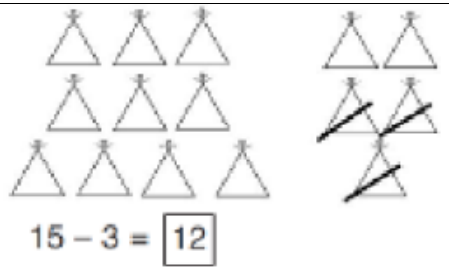
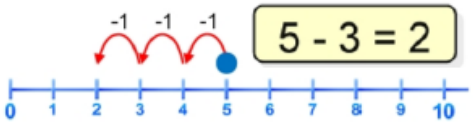
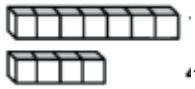
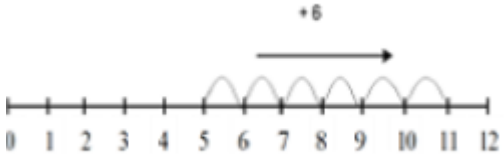
Stage 1 Practical (taking away)

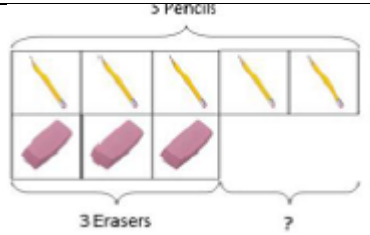
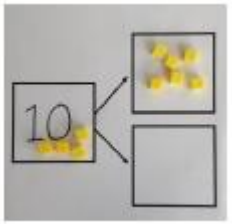
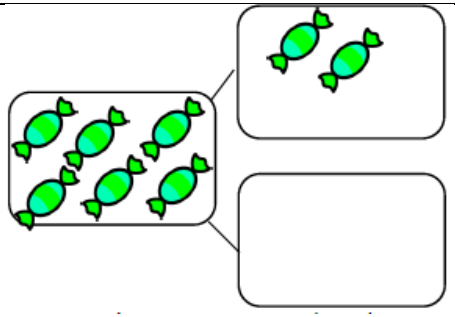
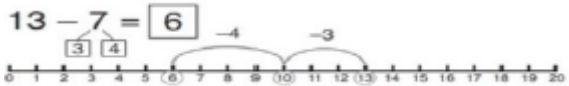
Prior to recording subtraction steps on a number line, children will work practically with equipment where they are 'taking away' a small group from a larger set of objects. As they become more confident, this practical subtraction will be mirrored on a number line so that the two are being done together. This will prepare them for the abstract concept of subtracting numbers rather than objects.

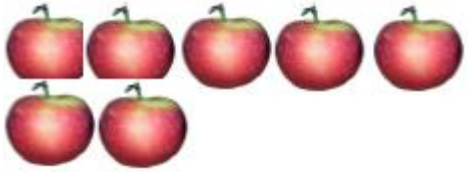
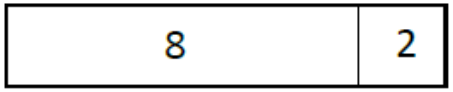
Counting back (to be introduced before counting up)

Steps in subtraction can be recorded from right to left on a number line. The steps often bridge through a multiple of 10 and, this is more efficient if children know how to partition 1-digit numbers.

Objective and Strategy	Concrete	Pictorial	Abstract

Taking away ones.	Use physical objects, counters, cubes etc to show how objects can be taken away.  6—4 = 2  4—2 = 2	 15 – 3 = 12 Cross out drawn objects to show what has been taken away.	7—4 = 3 16—9 = 7
Counting back	 Move objects away from the group, counting backwards.  Move the beads along the bead string as you count backwards.	 Count back in ones using a number line.	Put 13 in your head, count back 4. What number are you at?
Find the Difference	Compare objects and amounts  7 is 3 more than 4 I am 2 years older than my sister	Count on using a number line to find the difference.  	Hannah has 12 sweets and her sister has 5. How many more does Hannah have than her sister.?

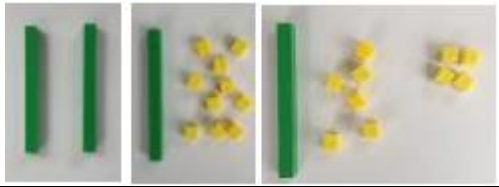
	 Lay objects to represent bar model		
Represent and use number bonds ad related subtraction facts within 20 Part, Part, Whole Model	 Link to addition. Use PPW model to model the inverse. If 10 is the whole and 6 is one of the parts, what is the other part? 10 – 6 = 4	 Use pictorial representation to sow the part.	 Move to using numbers within the part whole model.
Make 10	14 – 5  Make 14 on the ten frame. Take away to make ten, then take one more away so you have taken 5	13-7  Jump back 3 first, then another 4. Use ten as the stopping point.	16 – 8 How many do we take off first to get to 10? How many left to take off?

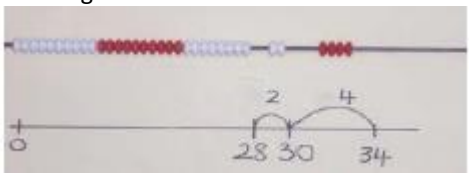
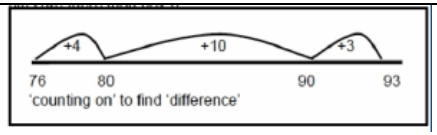
Bar Model	5- 2 =3 		 10= 8+2 10 = 2+8 10-2 =8 10-8 =2

Stage 2- Number tracks and number lines

Counting up (to be introduced after counting back)

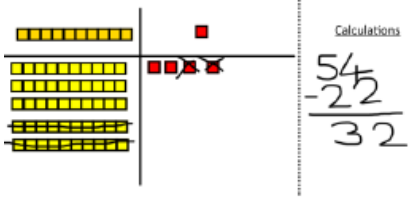
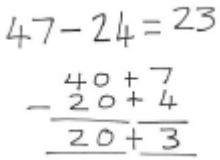
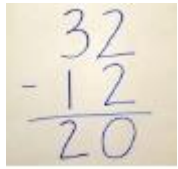
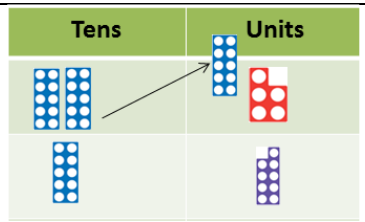
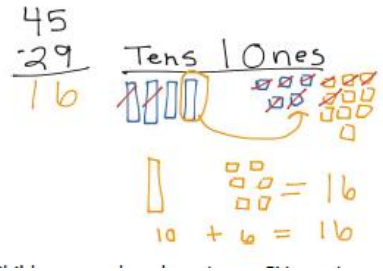
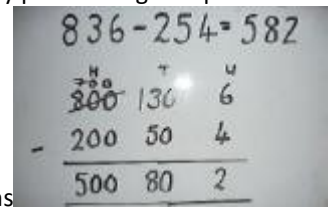
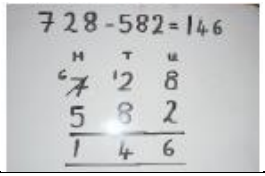
Steps in subtraction can be recorded from left to right on a number line. The steps often bridge through a multiple of 10

Objective and Strategy	Concrete	Pictorial	Abstract
Regroup a ten into ten ones	Use a Place Value chart to show how to change a ten into ten ones, use the term 'take and make' 	  20 – 4 =	20 – 4 = 16

Partitioning to sub-tract without re-grouping. *'Friendly numbers'*	 34—13 = 21 Use Dienes or numicon to show how to partition the number when subtracting without regrouping.	Students draw representations of Dienes or numicon and cross off.  43—21 = 22	43 – 21 = 22
Make ten strategy *Progression should be crossing one ten, crossing more than one ten, crossing the hundreds*	Use a bead bar or bead strings to model counting to next ten and the rest.  34 - 28	 Use a number line to count on to next ten and then the rest.	93—76 = 17

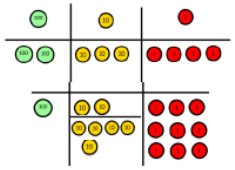
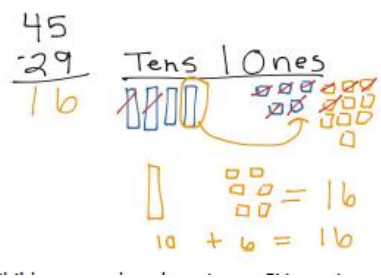
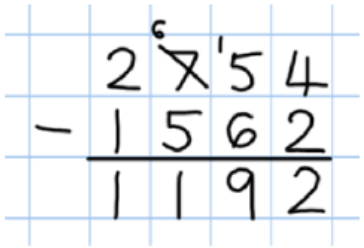
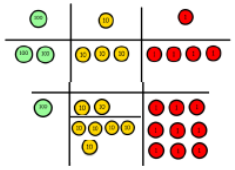
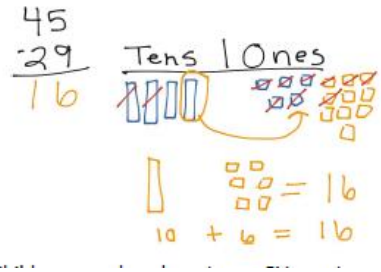
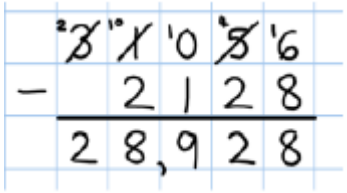
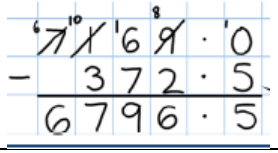
Stage 3: Partitioning (expanded columnar method)

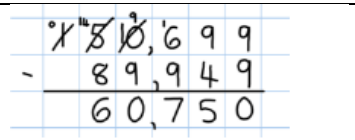
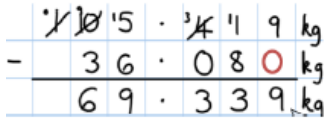
Partition both numbers into tens and units or hundreds, tens and units (using a grid makes this easier).

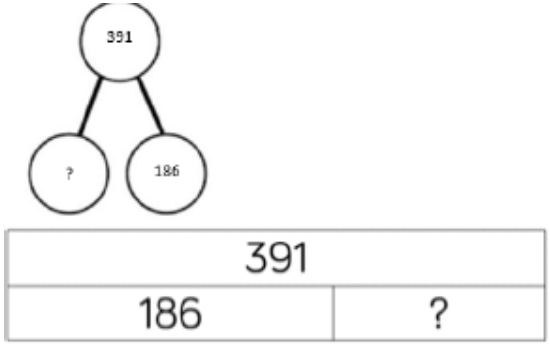
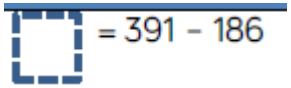
Objective and Strategy	Concrete	Pictorial	Abstract
Column subtraction without regrouping (friendly numbers)	47 – 32   Use base 10 or Numicon to model	 Draw representations to support understanding	 Intermediate step may be needed to lead to clear understanding. 
Column subtraction with regrouping	 Begin with bae 10 or Numicon. Move to place value counters, modelling the exchange of the ten into tens ones. Use the phrase ‘take and make’ for the exchange.	 Students may draw base ten or Place Value counters and cross off.	Begin by partitioning into place value columns  Then move onto formal method 

Stage 4 Efficient (Column method)

Column subtraction remains efficient when used with larger whole numbers or decimals, once learned, the method is quick and reliable.

Objective and Strategy	Concrete	Pictorial	Abstract
Subtracting tens and ones with up to 4 digits. *Introduce decimal subtraction through context of money*	234 - 179  Model process of exchange using Numicon, base ten and then move to Place Value counters.	Students may draw base ten or Place  Value counters and cross off.	 Use the phrase 'take and make' for exchange
Subtract with at least 4 digits, including money and measures. *Subtract with decimal values, including mixtures of integers and decimals and aligning the decimal*	234 – 179  Model process of exchange using Numicon, base ten and then move to Place Value counters.	Students may draw base ten or Place  Value counters and cross off.	 Use zeros for place holders 

Subtract with increasingly large and more complex numbers and decimal values.			 

Conceptual variation; different ways to ask children to solve 391 - 186			
	Raj spent £391, Timmy spent £186, how much more did Raj spend? Calculate the difference between 391 and 186	What is 186 less than 391? 	Missing digit values 

Multiplication

Written methods for Multiplication

It is important that children's mental methods of calculation are practised on a regular basis and secured alongside their learning and use of written methods of multiplication. The aim is that students use mental methods when appropriate, but for calculations that they cannot do in their heads they use a written method accurately and with confidence.

Students are taught and acquire secure mental methods of calculation and one written method of calculation for multiplication which they know they can rely on when mental methods are not appropriate. This policy shows the possible stages of each written method for multiplication, each stage building towards a more refined method.

There are some key basic skills that children need to help with multiplication, which include:

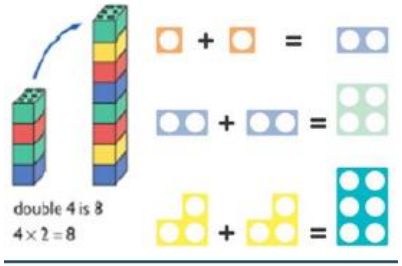
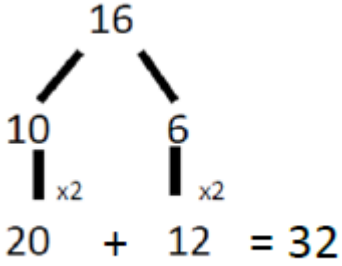
- counting
- estimating
- understanding multiplication as repeated addition
- recalling all multiplication facts to 12 × 12
- partitioning numbers into multiples of one hundred, ten and one
- working out products (70 × 5, 70 × 50, 700 × 5, 700 × 50) using the related fact 7 × 5 and their knowledge of place value
- adding two or more single-digit numbers mentally
- adding multiples of 10 (60 + 70) or of 100 (600 + 700) using the related addition fact, 6 + 7, and their knowledge of place value
- adding combinations of whole numbers
- understanding and using division and multiplication as inverse operations

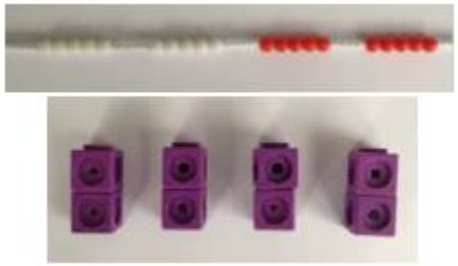
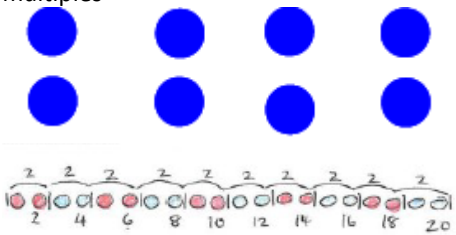
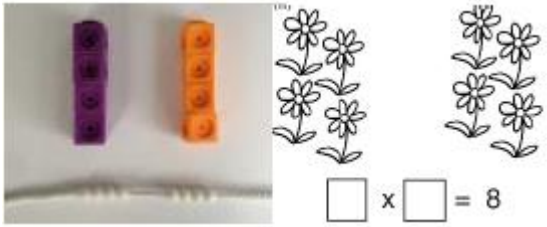
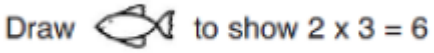
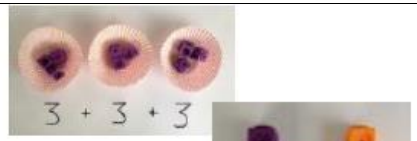
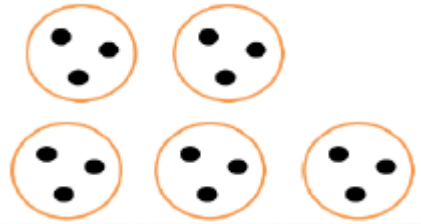
Using and applying is a key theme before students move onto the next stage in written calculation it is important that their skills are broadened through their use and application in a range of contexts, these include:

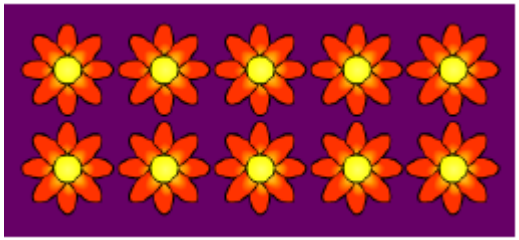
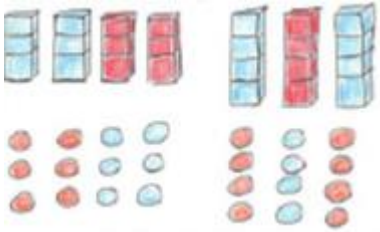
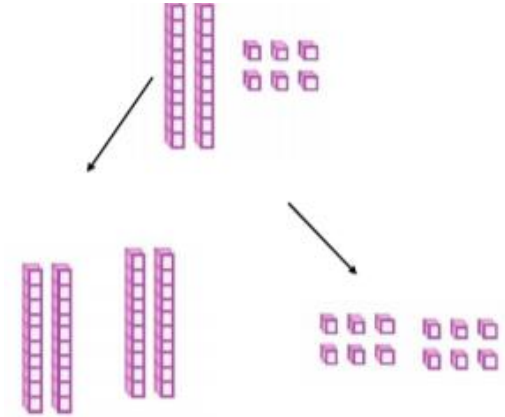
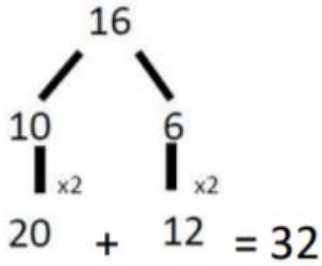
- using inverse
- missing box questions
- using units of measure including money and time
- word problems
- open ended investigations

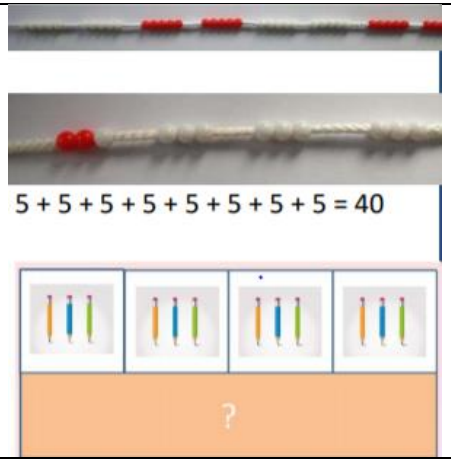
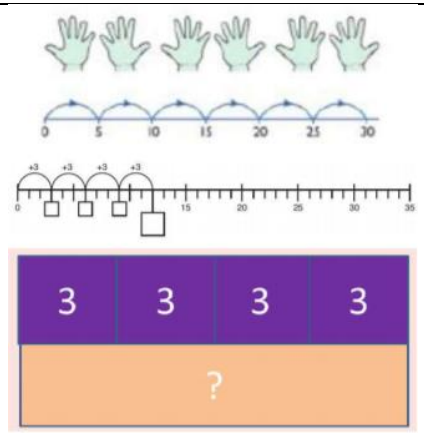
Stage 1: Practical (repeated addition)

Students will work practically with equipment grouping objects to see multiplication as repeated addition. As they become more confident, this practical grouping of objects will be mirrored on a number line using the vocabulary 'lots of', 'groups of', 'how many lots', 'how many times' so that the two are being done together. This will prepare them for the abstract concept of multiplying numbers rather than objects.

Objective and Strategy	**Concrete**	**Pictorial**	**Abstract**
Doubling	Use practical activities using manipulatives including cubes and numicons to demonstrate doubling 	Draw pictures to show how to double numbers Double 4 is 8 	Partition a number and then double each part before recombining it back together 

Counting in Multiples	Count the groups as students are skip counting, students may use their fingers as they are skip counting 	Students make representations to show multiples 	Count in multiples of a number aloud. Write sequences with multiples of numbers 2,4,6,8,10 5,10,15,20
Making equal groups and counting the total	Use manipulatives to create equal parts  	Draw and make representations  	2 x 4 = 8
Repeated addition	   Use different objects to add equal groups	Use pictorial number lines to solve problem. There are 3 sweets in one bag. How many sweets in 5 bags altogether?  3 +3 +3+ 3+ 3 = 15  2 +2+2+2+2 = 10	Write addition sentences to describe objects and pictures 

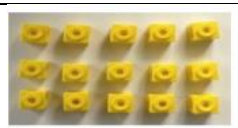
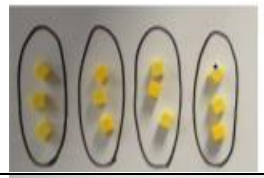
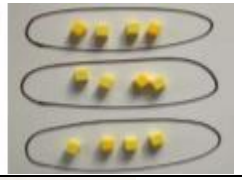
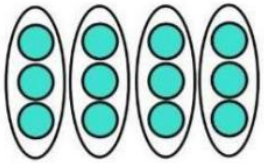
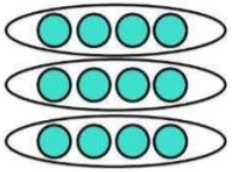
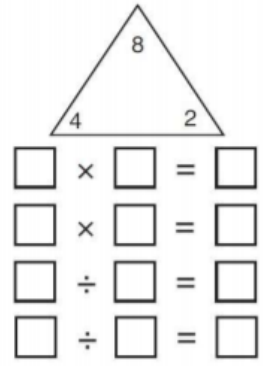
Understanding arrays	Use objects laid out in arrays to find the answer to 2 lots of 5, 3 lots of 3 etc 	Draw representations of arrays to show understanding 	3 x 2 = 6 2 x 5 = 10
Doubling	Model doubling using dienes, numicon and Place value counters  40 + 12 =52	Draw pictures and representations to show how to double numbers	Partition a number and then double each part before recombining it back together. 
Counting in multiples of 2,3,4,5,10 from 0 (repeated addition)	Count the groups as students are skip counting, students may use their fingers as they skip counting. Use bar models.	Number lines, counting sticks and bar models should be used to show representation of counting in multiples	Count in multiples of number aloud.

			Write sequences of multiples of numbers 0,2,4,6,8,10 0,3,6,9,12,15 0,5,10,15,20 4 x 3 = ☐

Stage 2 - Practical and pictorial arrays (towards grid method)

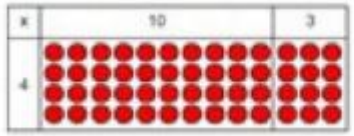
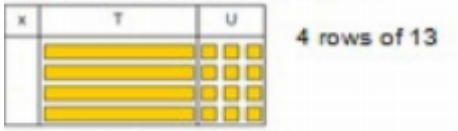
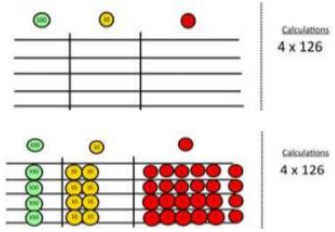
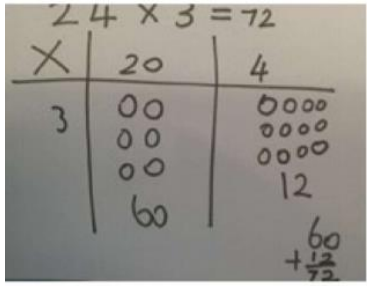
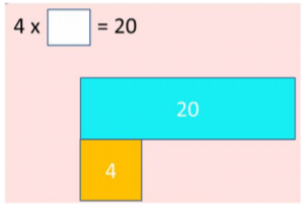
Students use arrays to demonstrate their understanding of commutativity for multiplication facts

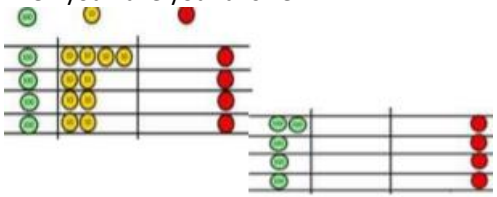
Objective and Strategy	Concrete	Pictorial	Abstract
Multiplication is commutative	Create arrays using counters and cubes and Numicon 	Use representation of arrays to show different calculations and explore commutativity.	12 =3 x 4 12= 4 x 3 Use an array to write multiplication sentences and reinforce repeated addition

	  Students should understand that an array can represent different equations and that, as multiplication is commutative, the order of multiplication does not affect the answer.  	 	 5 + 5 + 5 = 15 3 + 3 + 3 + 3 + 3 = 15 5 x 3 = 15 3 x 5 = 15
Using the inverse *This should be taught alongside division, so pupils learn how they work alongside each other.*			Show all 8 related fact family sentences 2 x 4 = 8 4 x 2 = 8 8 ÷ 2 = 4 8 ÷ 4 = 2 8 = 2 x 4 8 = 4 x 2 2 = 8 ÷ 4 4 = 8÷ 2

Stage 3 Partitioning Grid method

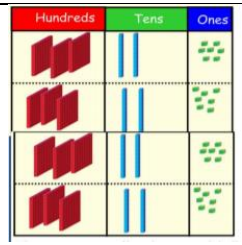
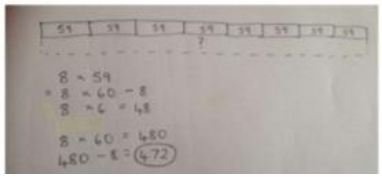
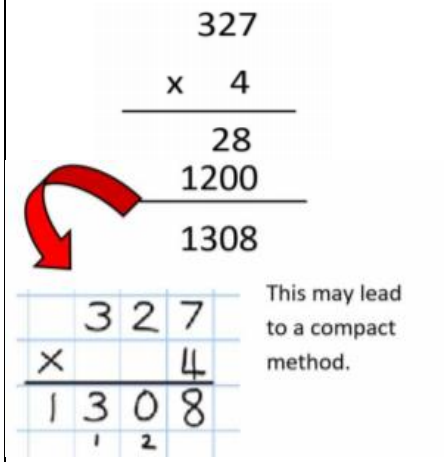
Students should be able to recall and use multiplication facts for the 3,4, and 8 times tables

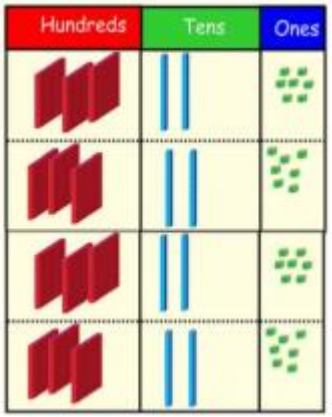
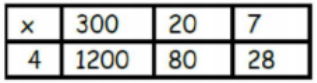
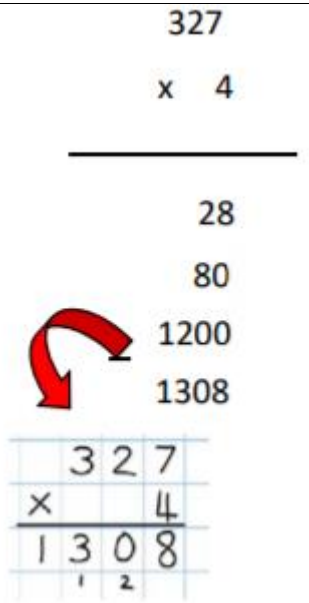
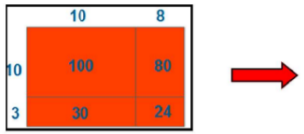
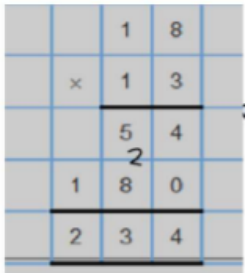
Objective and Strategy	Concrete	Pictorial	Abstract
Grid method, progressing to formal method Multiply 2 digits by 1 digit numbers	Show the links with arrays to first introduce the grid method  Move onto base ten to move towards a more compact method  Move onto place value counters to show how we are finding groups of a number. We are multiplying 4 so we need 4 rows  Fill each row with 126	Students can represent their work with place value counters in a way that they understand. They can draw the counters using colours to show the different columns to show their thinking as shown below  Bar model are used to explore missing numbers 	Start with multiplying by one digit numbers and showing the clear addition alongside the grid **×** | **30** | **5** **7** | **210** | **35** **210 + 35 = 245** Move forward to the formal written method: 35 X 7 245 3

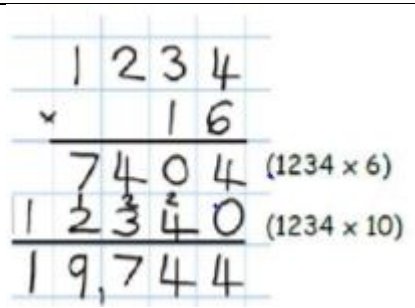
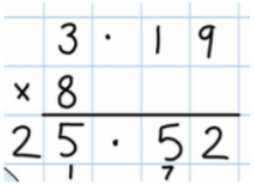
	Add each column, starting with the ones making any exchanges needed. Then you have your answer. 		
Solve problems including missing number problems, integer scaling problems			Three times as high, eight times as long ? x 5 = 20 20 ÷ ? = 5 3 hats and 4 coats how many different outfits?

Stage 4 Short (column)

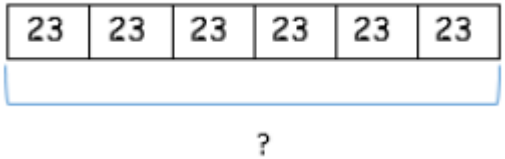
Objective and Strategy	Concrete	Pictorial	Abstract
Column multiplication	Students can continue to be supported by place value counters and numicon at the stage of multiplication. This is initially done where there is no regrouping 321 x 2 = 642	The grid method may be used to show how this relates to a formal written method × | 300 | 20 | 7 4 | 1200 | 80 | 28	

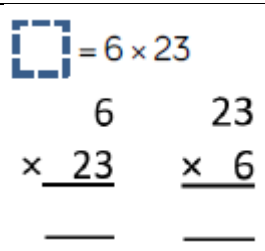
	 It is important at this stage that they always multiply the ones first The corresponding long multiplication is modelled alongside	Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal methods 	

Column multiplication for 3 and 4 digits x 1 digit.	It is important at this stage that they always multiply the ones first. Students can continue to be supported by place value counters and numicon at this stage of multiplication. This is initially done where there is no regrouping  321 x 2 = 642	 	
Column Multiplication	Manipulatives may still be used with the corresponding long multiplication modelled alongside	Continue to use bar modelling to support problem solving  	 18 x 3 on the first row (8 x3 =24, carrying the 2 for 20 then 1 x 3) 18 x 10 0n the second row. Show multiply by 10 by putting zero in units first.

			
Multiply decimals up to 2 decimal places by a single digit.			Remind students that the single digits belongs to the units column, Line up the decimal points in the question and answer 

Conceptual variation: different ways to ask students to solve 6 x 23

	Mai had to swim 23 lengths, 6 times a week. How many lengths did she swim in one week? With counters prove that 6 x 23 = 138	Find the product of 6 and 23 6 x 23 =	What is the calculation? What is the product? 

Division

Written methods for Division

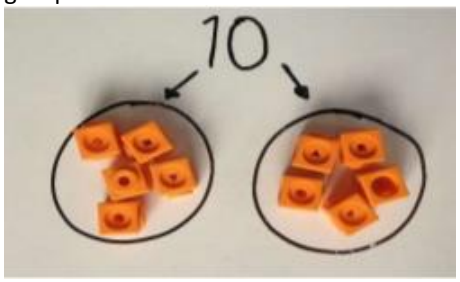
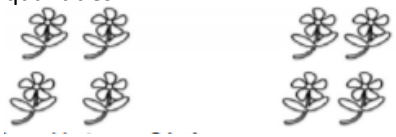
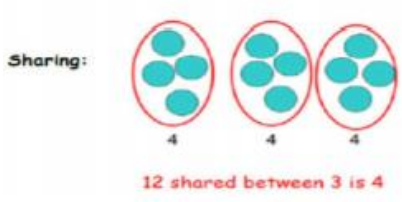
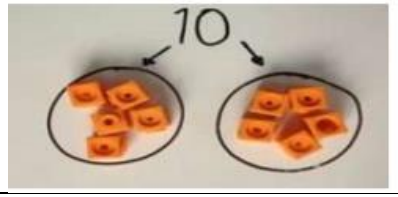
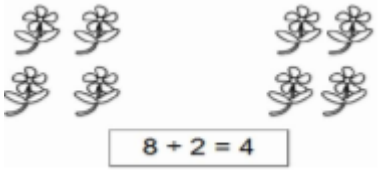
It is important that student's mental methods of calculation are practised on a regular basis and secured alongside their learning and use of written methods of division. The aim is that students use mental methods when appropriate, but for calculations that they cannot do in their heads they use a written method accurately and with confidence. Students are taught and acquire secure mental methods of calculation and one written method of calculation for division which they know they can rely on when mental methods are not appropriate. This policy shows the possible stages of each written method for division, each stage building towards a more refined method. There are some key basic skills that children need to help with subtraction, which include:

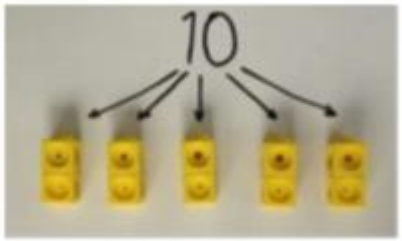
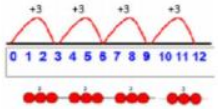
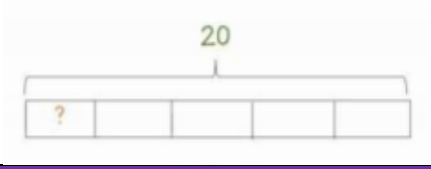
- counting
- estimating
- understanding division as repeated subtraction

• partitioning two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways (432 into 400 + 30 + 2 and also into 300 + 120 + 12) • recalling multiplication and division facts to 12 × 12
• recognising multiples of one-digit numbers and dividing multiples of 10 or 100 by a single-digit number using their knowledge of division facts and place value
• knowing how to find a remainder working mentally, for example, find the remainder when 48 is divided by 5
• understanding and using division and multiplication as inverse operations.

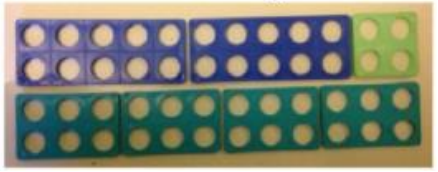
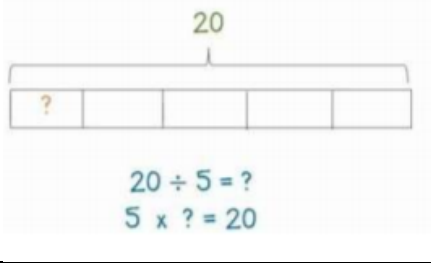
Using and applying is a key theme and one of the aims of National Curriculum and before students move onto the next stage in written calculation it is important that their skills are broadened through their use and application in a range of contexts, these include:
• using inverse
• missing box questions
• using units of measure including money and time
• word problems
• open ended investigations

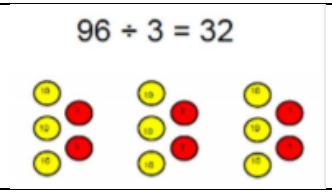
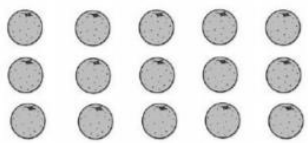
Stage 1- Sharing			
Objective and Strategy	**Concrete**	**Pictorial**	**Abstract**

Division as sharing	  I have 10 cubes, can you share them equally in 2 groups? 	Students use pictures or shapes to share quantities  8 shared between 2 is 4 	12 shared between 3 is 4
Division as sharing	I have 10 cubes, can you share equally in 2 groups? 	Students use pictures or shapes to share quantities 	12 ÷ 3 = 4

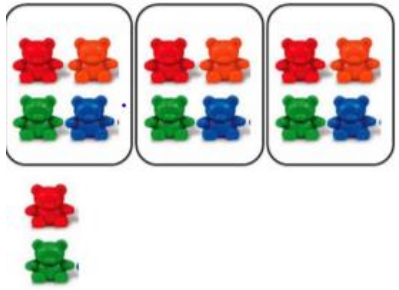
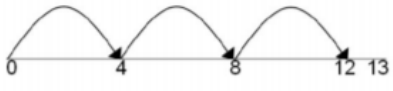
		Students use bar modelling to show and support understanding  12 ÷ 3 = 4	
Division as grouping	Divide quantities into equal groups. Use cubes, counters, objects numicon or place value counters to aid understanding.   	Use number lines for grouping  12 ÷ 3 = 4 Think of a bar a whole. Split it into the number of groups you are dividing by and work out how many would be in each group.  	28÷7 =4 Divide 28 into 7 groups. How many are in each group?

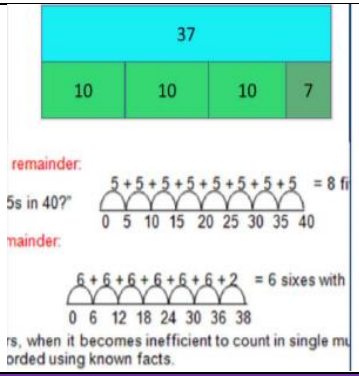
Stage 2 – (Grouping)

Objective and Strategy	Concrete	Pictorial	Abstract
Division as grouping	Use cubes, counters, objects, place value counters or numicon to aid understanding.  24 divided into groups of 6 = 4	Continue to use bar modelling to aid solving division problems.  	How many groups of 6 in 24? 24 ÷ 6 = 4

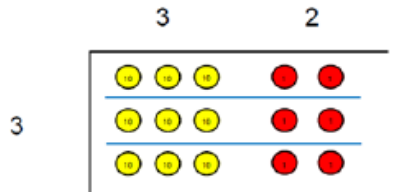
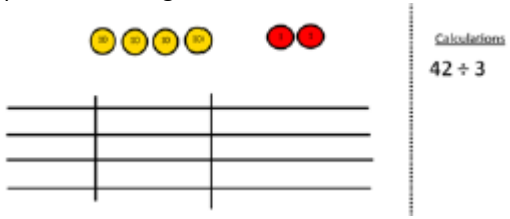
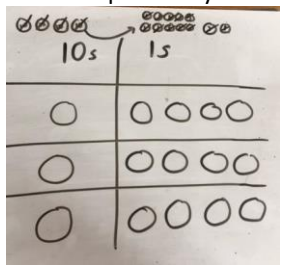
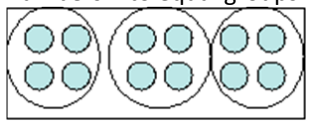
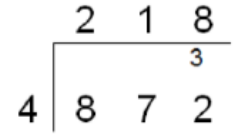
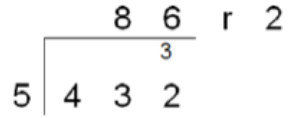
			
Division with arrays	Link division to multiplication by creating an array and thinking about the number sentences that can be created eg:  15 ÷ 3= 5 5 x 3 = 15 15 ÷5 = 3 3 x 5 = 15	Draw an array and use lines to split the array into groups to make multiplication and division sentences 	Find the inverse of multiplication and division sentences by creating eight linking number sentences 7 7 x 4= 28 28 ÷ 7 = 4 28 ÷ 4 = 7 28 = 4 x 7 4 = 28 ÷ 7 7= 28÷ 4

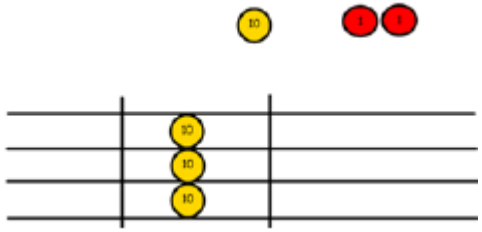
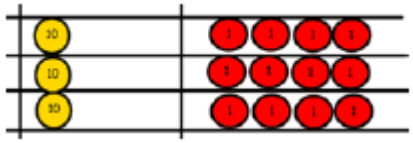
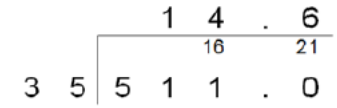
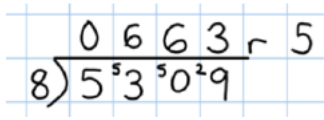
Stage 3 – Division with remainders

Objective and Strategy	Concrete	Pictorial	Abstract
Division with remainders	14 ÷ 3 = Divide objects between groups and see how much is left over 	Jump forward in equal jumps on the number line then see how many more you need to jump to find a remainder  Draw dots and group them to divide an amount and clearly show a reminder.  Use bar models to show division with remainders	Complete written divisions and show the remainder using r. 

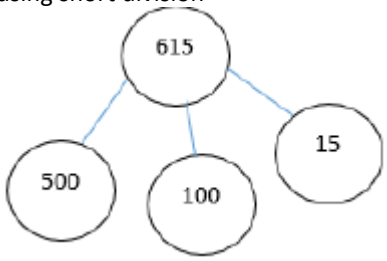
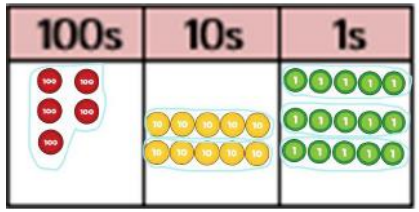
			

Stage 3 – Short division (Bus stop method)

Objective and Strategy	Concrete	Pictorial	Abstract
Divide at least 3 digit number by 1 digit Short division	96÷3 tens units  Use place value counters to divide using the bus stop method along side  42 ÷ 3 Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten into each group and we have 1 ten left over.	Students to represent the place value counters pictorially.  Students can continue to draw diagrams with dots or circles to help divide numbers into equal groups 	Begin with divisions that divide equally with no remainder  Move onto divisions with a remainder  Finally move onto decimals

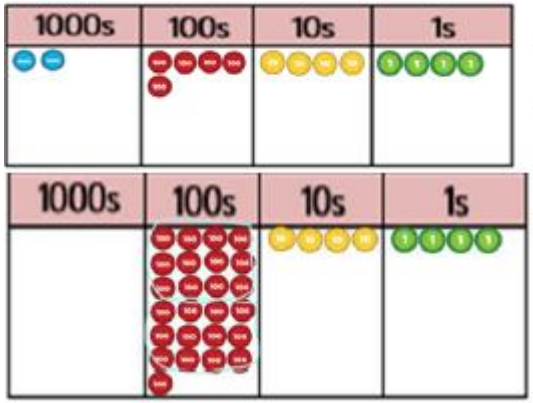
	 We exchange this ten for ones and the share equally among the groups.  We look how much is in one group so the answer is 14.		 

Conceptual variation; different ways to ask children to solve 615 ÷ 5

Using part of a whole model below, how can you divide 615 by 5 without using short division  	I have £615 and share it equally between 5 bank accounts. How much is in each account? 615 pupils need to be put into 5 groups. How many will be in each group?	$5 \overline{)615}$ $615 \div 5 =$ $\square = 615 \div 5$	What is the calculation? What is the answer? 

Stage 4 – Long division

Long division using place value counters 2544 ÷12



We can’t group 2 thousand into groups of 12 so will exchange them

We can group 24 hundreds into groups of 12 which leaves with 1 hundreds

$$\begin{array}{r} 02 \\ 12\overline{)2544} \\ \underline{24} \\ 1 \end{array}$$

Step 1 long division – remainder in the ones

h t o

0 4 1 R1

4 does not go into 1(hundred). So combine 1 hundred with 6 tens (160)
4 goes into 16 four times
4 goes into 5 once leaving a remainder of 1

th h t o

$$\begin{array}{r} 0400R7 \\ 8\overline{)3207} \end{array}$$

8 does not go into 3 of the thousands. SO combine the 3 thousands and the 2 hundreds (3,200)
8 goes into 32 four times (3200 ÷ 8 = 400)
8 goes into 0 zero times (tens)
8 goes into 7 zero times and leaves a remainder of 7
When dividing the ones, 4 goes into 7 one time, multiply 1 x 4 = 4 write that four under the 7 and subtract. This finds us the remainder of 3.

h t o

$$\begin{array}{r} 061 \\ 4\overline{)247} \\ -4 \\ \hline 3 \end{array}$$

Check 4 x 61 + 3 = 247

th h t o 0 4 0 2 4) 1 6 0 9 − 8 1	When dividing the ones, 4 goes into 9 two times. Multiply 2 x 4 = 8, wright eight under 9 and subtract. This finds us the remainder of 1 Check 4 x 402 + 1 = 1,609
Step 2 – Long multiplication	

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
t o 2 2)5 8	t o 2 2)5 8 - 4 1	t o 2 9 2)5 8 - 4 ↓ 1 8
Two goes into 5 two times, or 5 tens ÷ 2 = 2 whole tens -- but there is a remainder!	To find it, multiply 2 × 2 = 4, write that 4 under the five, and subtract to find the remainder of 1 ten.	Next, drop down the 8 of the ones next to the leftover 1 ten. You combine the remainder ten with 8 ones, and get 18.

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
t o 2 9 2)5 8 - 4 1 8	t o 2 9 2)5 8 - 4 1 8 - 1 8 0	t o 2 9 2)5 8 - 4 1 8 - 1 8 0
Divide 2 into 18. Place 9 into the quotient.	Multiply 9 × 2 = 18, write that 18 under the 18, and subtract.	The division is over since there are no more digits in the dividend. The quotient is 29.

A remainder in any place value

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
h t o 1 2)2 7 8 Two goes into 2 one time, or 2 hundreds ÷ 2 = 1 hundred.	h t o 1 2)2 7 8 - 2 0 Multiply 1 × 2 = 2, write that 2 under the two, and subtract to find the remainder of zero.	h t o 1 8 2)2 7 8 - 2 ↓ 0 7 Next, drop down the 7 of the tens next to the zero.
Divide.	**Multiply & subtract.**	**Drop down the next digit.**
h t o 1 3 2)2 7 8 - 2 0 7 Divide 2 into 7. Place 3 into the quotient.	h t o 1 3 2)2 7 8 - 2 0 7 - 6 1 Multiply 3 × 2 = 6, write that 6 under the 7, and subtract to find the remainder of 1 ten.	h t o 1 3 2)2 7 8 - 2 0 7 - 6 1 8 Next, drop down the 8 of the ones next to the 1 leftover ten.
1. Divide.	**2. Multiply & subtract.**	**3. Drop down the next digit.**
h t o 1 3 9 2)2 7 8 - 2 0 7 - 6 1 8 Divide 2 into 18. Place 9 into the quotient.	h t o 1 3 9 2)2 7 8 - 2 0 7 - 6 1 8 - 1 8 0 Multiply 9 × 2 = 18, write that 18 under the 18, and subtract to find the remainder of zero.	h t o 1 3 9 2)2 7 8 - 2 0 7 - 6 1 8 - 1 8 0 There are no more digits to drop down. The quotient is 139.